Implementation of the BDT Model with Different Volatility Estimators: Applications to Eurodollar Futures Options

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This article concentrates on the effects of different classes of volatility estimators in the pricing of interest rate-sensitive options using the single-factor Black, Derman, and Toy [1990] (henceforth BDT) model. The short-term interest rate that drives the changes in the entire term structure is the fundamental variable in the BDT model. Therefore, the choice of a class of volatility estimation model for short rates is crucial for pricing interest-sensitive claims.

Since both the current level and the stochastic properties of volatility affect the distribution of future interest rate levels, which, in turn, determine the price of the derivative instrument, we model both the instantaneous and time series properties of the interest rate volatility in pricing Eurodollar futures options. One of our contributions is to provide further empirical evidence that using a more accurate representation of the volatility structure reduces pricing errors.

Empirical results, based on 4,228 estimated prices, indicate that valuation of Eurodollar futures options is sensitive to the volatility model used. We employ *moving average* models, such as constantly weighted (CWMA) and exponentially weighted moving average (EWMA), and time series models, such as generalized autoregressive conditional heteroscedasticity (GARCH) and integrated GARCH (IGARCH), in estimating the volatility of short rates. Our intention is to determine which of the volatility estimators best captures the dynamics of short rates and best predicts the prices of Eurodollar futures options.

The predictive power of the volatility estimation models is tested using the mean square error (MSE) and the Theil inequality coefficient (TIC) when the estimated prices are compared to the actual prices. The results indicate that, within the BDT framework, the forecasting ability of the moving average volatility estimators is inferior to that of the time series volatility models.

To price interest rate-contingent claims, academicians and practitioners use both *equilibrium* and *arbitrage-free* models.[1] In an equilibrium model, the initial term structure is an output from the model, while in an arbitrage-free model, the initial term structure of *yields* and their estimated *volatilities* are inputs to the model.

The effect of using different volatility estimators on pricing interest rate derivatives in the equilibrium models has been extensively studied in the literature (see Marsh and Rosenfeld [1983], Barone, Cuoco, and Zautzik [1991], Chan et al. [1992], Longstaff and Schwartz [1992], Brenner, Harjes, and Kroner [1996], and Nowman [1997]). Earlier studies that deal with pricing interest-sensitive claims using arbitrage-free models do not test the effect of different volatility estimation models on derivatives pricing (e.g., Kalotay, Williams, and Fabozzi [1993], and